NAG Toolbox for MATLAB

g13cf

1 Purpose

For a bivariate time series, g13cf calculates the gain and phase together with lower and upper bounds from the univariate and bivariate spectra.

2 Syntax

```
[gn, gnlw, gnup, ph, phlw, phup, ifail] = g13cf(xg, yg, xyrg, xyig,
stats, 'ng', ng)
```

3 Description

Estimates of the gain $G(\omega)$ and phase $\phi(\omega)$ of the dependency of series y on series x at frequency ω are given by

$$\hat{G}(\omega) = \frac{A(\omega)}{f_{xx}(\omega)}$$

$$\hat{\phi}(\omega) = \arccos\left(\frac{cf(\omega)}{A(\omega)}\right), \qquad \text{if } qf(\omega) \geq 0$$

$$\hat{\phi}(\omega) = 2\pi - \arccos\left(\frac{cf(\omega)}{A(\omega)}\right), \quad \text{if } qf(\omega) < 0.$$

The quantities used in these definitions are obtained as in Section 3 of the document for g13ce.

Confidence limits are returned for both gain and phase, but should again be taken as very approximate when the coherency $W(\omega)$, as calculated by g13ce, is not significant. These are based on the assumption that both $\left(\hat{G}(\omega)/G(\omega)\right) - 1$ and $\hat{\phi}(\omega)$ are Normal with variance

$$\frac{1}{d}\left(\frac{1}{W(\omega)} - 1\right).$$

Although the estimate of $\phi(\omega)$ is always given in the range $[0, 2\pi)$, no attempt is made to restrict its confidence limits to this range.

4 References

Bloomfield P 1976 *Fourier Analysis of Time Series: An Introduction* Wiley

Jenkins G M and Watts D G 1968 *Spectral Analysis and its Applications* Holden–Day

5 Parameters

5.1 Compulsory Input Parameters

1: **xg(ng) – double array**

The **ng** univariate spectral estimates, $f_{xx}(\omega)$, for the x series.

2: **yg(ng) – double array**

The **ng** univariate spectral estimates, $f_{yy}(\omega)$, for the y series.

3: **xyrg(ng) – double array**

The real parts, $cf(\omega)$ of the **ng** bivariate spectral estimates for the x and y series. The x series leads the y series.

4: **xyig(ng) – double array**

The imaginary parts, $qf(\omega)$, of the **ng** bivariate spectral estimates for the x and y series. The x series leads the y series.

Note: the two univariate and the bivariate spectra must each have been calculated using the same method of smoothing. For rectangular, Bartlett, Tukey or Parzen smoothing windows, the same cut-off point of lag window and the same frequency division of the spectral estimates must be used. For the trapezium frequency smoothing window, the frequency width and the shape of the window and the frequency division of the spectral estimates must be the same. The spectral estimates and statistics must also be unlogged.

5: **stats(4) – double array**

The four associated statistics for the univariate spectral estimates for the x and y series. **stats**(1) contains the degrees of freedom, **stats**(2) and **stats**(3) contain the lower and upper bound multiplying factors respectively and **stats**(4) holds the bandwidth.

Constraint: **stats**$(1) \geq 3.0$.

5.2 Optional Input Parameters

1: **ng – int32 scalar**

Default: The dimension of the arrays **xg**, **yg**, **xyrg**, **xyig**, **gn**, **gnlw**, **gnup**, **ph**, **phlw**, **phup**. (An error is raised if these dimensions are not equal.)

the number of spectral estimates in each of the arrays **xg**, **yg**, **xyrg** and **xyig**. It is also the number of gain and phase estimates.

Constraint: **ng** ≥ 1.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **gn(ng) – double array**

The **ng** gain estimates, $\hat{G}(\omega)$, at each frequency ω.

2: **gnlw(ng) – double array**

The **ng** lower bounds for the **ng** gain estimates.

3: **gnup(ng) – double array**

The **ng** upper bounds for the **ng** gain estimates.

4: **ph(ng) – double array**

The **ng** phase estimates, $\hat{\phi}(\omega)$, at each frequency ω.

5: **phlw(ng) – double array**

The **ng** lower bounds for the **ng** phase estimates.

6: **phup(ng) – double array**

The **ng** upper bounds for the **ng** phase estimates.

7: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail = 1

On entry, **ng** < 1,
or **stats**(1) < 3.0.

ifail = 2

A bivariate spectral estimate is zero. For this frequency the gain and the phase and their bounds are set to zero.

ifail = 3

A univariate spectral estimate is negative. For this frequency the gain and the phase and their bounds are set to zero.

ifail = 4

A univariate spectral estimate is zero. For this frequency the gain and the phase and their bounds are set to zero.

ifail = 5

A calculated value of the squared coherency exceeds 1.0. For this frequency the squared coherency is reset to 1.0 in the formulae for the gain and phase bounds.

If more than one failure of types 2, 3, 4 and 5 occurs then the failure type which occurred at lowest frequency is returned in **ifail**. However the actions indicated above are also carried out for failures at higher frequencies.

7 Accuracy

All computations are very stable and yield good accuracy.

8 Further Comments

The time taken by g13cf is approximately proportional to **ng**.

9 Example

```
xg = [2.0349;
      0.51554;
      0.0764;
      0.01068;
      0.0009300000000000001;
      0.001;
      0.00076;
      0.00037;
      0.00021];
yg = [21.97712;
```

```
     3.29761;
     0.28782;
     0.0248;
     0.00285;
     0.00203;
     0.00125;
     0.00107;
     0.00191];
xyrg = [-6.54995;
     0.34107;
     0.12335;
     -0.00514;
     -0.00033;
     -0.00039;
     -0.00026;
     0.00011;
     6.999999999999999e-05];
xyig = [0;
     -1.1903;
     0.04087;
     0.00842;
     0.00032;
     -1e-05;
     0.00018;
     -0.00016;
     0];
stats = [30;
     0.63858;
     1.7867;
     0.33288];
[gn, gnlw, gnup, ph, phlw, phup, ifail] = g13cf(xg, yg, xyrg, xyig,
stats)
```

```
gn =
    3.2188
    2.4018
    1.7008
    0.9237
    0.4943
    0.3901
    0.4161
    0.5248
    0.3333
gnlw =
    2.9722
    2.1138
    1.3748
    0.5558
    0.1327
    0.1002
    0.1346
    0.1591
    0.0103
gnup =
    3.4859
    2.7290
    2.1042
    1.5350
    1.8415
    1.5196
    1.2863
    1.7306
   10.8301
ph =
    3.1416
    4.9915
    0.3199
    2.1189
    2.3716
    3.1672
```

```
    2.5360
    5.3147
         0
phlw =
    3.0619
    4.8637
    0.1071
    1.6109
    1.0563
    1.8075
    1.4074
    4.1214
   -3.4809
phup =
    3.2213
    5.1192
    0.5328
    2.6268
    3.6868
    4.5270
    3.6647
    6.5079
    3.4809
ifail =
           0
```